

Chapt. 9

9.47-

Fig 9.15 Analysis of Variance for time

Service	DF	SS	MS	F	P
display	2 (p-1)	500.17	250.08	30.1	0.000
Error	9 (n-p)	74.75	8.31		
Total	11 (n-1)	574.92			

- a. $H_0: \mu_A = \mu_B = \mu_C$ (All treatment means are equal)
 H_a : at least two of μ_A, μ_B, μ_C differ (at least two treatment means differ)
 Define the statistic

$$F = \frac{MST}{MSE} = \frac{SST/(p-1)}{SSE/(n-p)} = \frac{250.08}{8.3056} = 30.11$$

$$F = 30.11 > F(2, 9)_{0.05} = 4.26, \quad p\text{-value} \approx 0.000$$

Reject $H_0: \mu_A = \mu_B = \mu_C$; at least two displays differ.

- b. A Tukey Simultaneous $100(1-\alpha)\%$ confidence interval for $\mu_i - \mu_h$ is

$$\left[(\bar{x}_i - \bar{x}_h) \pm t_{(p, n-p), 0.05} \sqrt{\frac{MSE}{m}} \right], \quad 95\% \text{ confidence interval} \quad \begin{cases} n=12 \\ p=3 \\ n-p=9 \\ m=4 \end{cases}$$

$$\mu_A - \mu_B = 24.5 - 20.5 = 4.0; \quad \left[4.0 \pm 3.95 \sqrt{\frac{8.31}{4}} \right] \rightarrow [4 \pm 5.69] \rightarrow \underline{[-1.69, 9.69]}$$

$$\mu_A - \mu_C = 24.5 - 35.75 = -11.25; \quad [-11.25 \pm 5.69] \rightarrow \underline{[-16.94, -5.56]}$$

$$\mu_B - \mu_C = 20.5 - 35.75 = -15.25; \quad [-15.25 \pm 5.69] \rightarrow \underline{[-20.94, -9.56]}$$

B appears to Min the time

- c. An individual $100(1-\alpha)\%$ confidence interval for $\mu_i - \mu_h$ is:

$$\left[(\bar{x}_i - \bar{x}_h) \pm t_{(n-p), \frac{\alpha}{2}} \sqrt{MSE \left(\frac{1}{n_i} + \frac{1}{n_h} \right)} \right] \quad \begin{matrix} n_i=4 & n_h=4 \\ n=12, & p=3 \end{matrix}$$

$$\mu_A - \mu_B = 24.5 - 20.5 = 4.0; \quad \left[4 \pm t_{9, 0.05} \sqrt{8.31 \left(\frac{1}{4} + \frac{1}{4} \right)} \right] = [4 \pm 4.6128] = \underline{[-0.61, 8.61]}$$

$$\mu_A - \mu_C = 24.5 - 35.75 = -11.25; \quad [-11.25 \pm 4.6128] = \underline{[-15.86, -6.64]}$$

$$\mu_B - \mu_C = 20.5 - 35.75 = -15.25; \quad [-15.25 \pm 4.6128] = \underline{[-19.86, -10.64]}$$

10.17

a. $n=1000$, $P_{GM} = 36\%$, $P_{Jap.} = 26\%$, $P_{Ford} = 21\%$, $P_{Chrysler} = 9\%$, $P_{Other} = 8\%$

$E_i = np_i$ at least ≥ 5

$$E_{GM} = 1000 \times 36\% = 360 \geq 5$$

$$E_{Jap.} = 1000 \times 26\% = 260 \geq 5$$

$$E_{Ford} = 1000 \times 21\% = 210 \geq 5$$

$$E_{Chrysler} = 1000 \times 9\% = 90 \geq 5$$

$$E_{Other} = 1000 \times 8\% = 80 \geq 5$$

All E_i Value ≥ 5 , So it is appropriate to carry out a chi-square test.

b.
$$\chi^2 = \sum_{i=1}^k \frac{(f_i - E_i)^2}{E_i}$$

$\begin{cases} H_0 = \text{Current market share not differ from those of 1990.} \\ H_a = \text{Current market share differ from those of 1990.} \end{cases}$

$$= \frac{(291-360)^2}{360} + \frac{(202-260)^2}{260} + \frac{(225-210)^2}{210} + \frac{(53-90)^2}{90} + \frac{(29-80)^2}{80}$$
$$= 2.6694 + 12.9385 + 20.1190 + 15.2111 + 9.0125$$
$$= 50.9505$$

$\chi_{(n-1), \alpha}^2 \Rightarrow \chi_{(5-1), 0.05}^2$ 經由 P594 查 table A.10 表得知為 9.48773

因 $\chi^2 > \chi_{\alpha}^2 \Rightarrow 50.9505 > 9.48773$ 故

Reject $H_0 = p_1 = 0.36, p_2 = 0.26, p_3 = 0.21, p_4 = 0.09, p_5 = 0.08$

10.29

$$a. \chi^2 = \sum_{\text{all cell}} \frac{(f_{ij} - \hat{E}_{ij})^2}{\hat{E}_{ij}} = \frac{(15 - \frac{40 \times 26}{78})^2}{\frac{40 \times 26}{78}} + \frac{(0 - \frac{40 \times 26}{78})^2}{\frac{40 \times 26}{78}} + \frac{(25 - \frac{40 \times 26}{78})^2}{\frac{40 \times 26}{78}} + \frac{(1 - \frac{26 \times 35}{78})^2}{\frac{26 \times 35}{78}} + \frac{(25 - \frac{26 \times 35}{78})^2}{\frac{26 \times 35}{78}} + \frac{(0 - \frac{26 \times 35}{78})^2}{\frac{26 \times 35}{78}} + \frac{(1 - \frac{3 \times 26}{78})^2}{\frac{3 \times 26}{78}} + \frac{(1 - \frac{3 \times 26}{78})^2}{\frac{3 \times 26}{78}} + \frac{(1 - \frac{3 \times 26}{78})^2}{\frac{3 \times 26}{78}}$$

$\left\{ \begin{array}{l} H_0: \text{independence} \\ H_a: \text{not independence} \end{array} \right.$

$= 0.2083 + 13.3333 + 10.2084 + 0 + 0 + 0 + 0.2381 + 15.2380 + 11.6667$

$= 50.8929$

$\alpha = 0.05, r = 3, c = 3, df = 4$

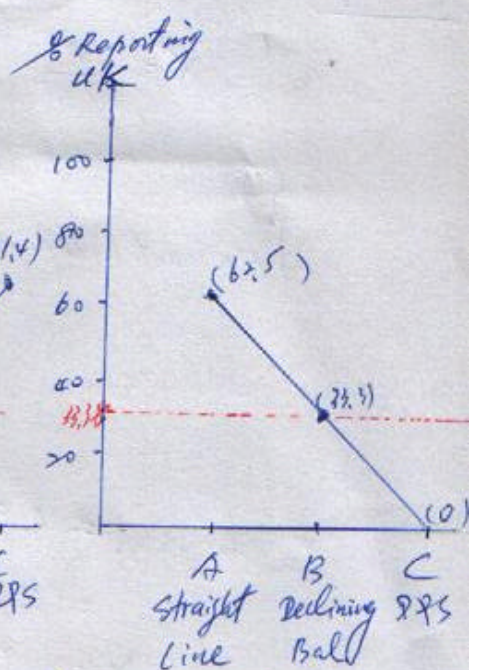
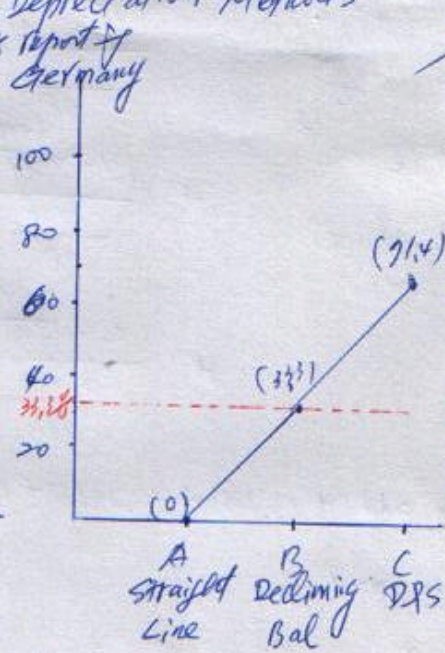
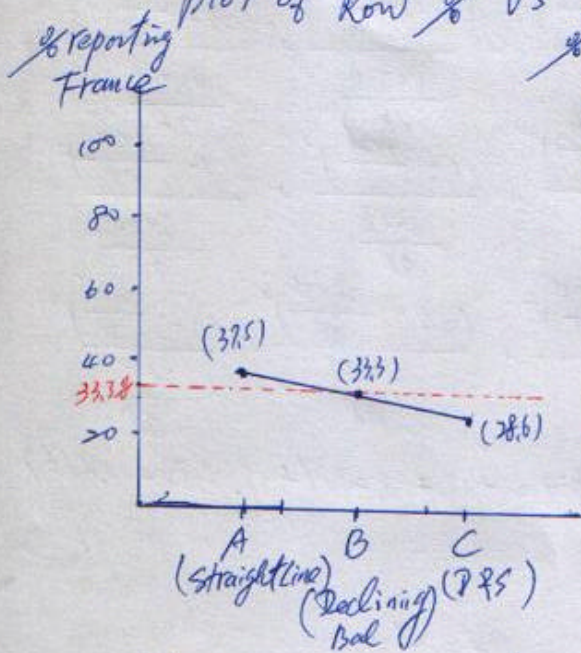
$\chi^2_{(r-1)(c-1), 0.05}$ 查表 p594 Table A.10 查表 $\chi^2_{4, 0.05} = 9.48773$

因 $\chi^2 > \chi^2_{4, 0.05}$ (即 $50.8929 > 9.4877$) 故

Reject $H_0 = \text{independence}$

Depreciation Methods	France	Germany	UK	total	
A. straight line(s)	15	0	25	40	
% of row	37.5%	0	62.5%	100%	
% of column	57.2%	0	96.2%	51.3%	
% of total	19.2%	0	32.1%	51.3%	
B. Declining Bal(D)	1	1	1	3	
% of row	33.3%	33.3%	33.3%	100%	
% of column	3.8%	3.8%	3.8%	3.8%	
% of total	1.3%	1.3%	1.3%	3.8%	
C. (D&S)	10	25	0	35	
% of row	28.6%	71.4%	0	100%	
% of column	38.5%	96.2%	0	44.9%	
% of total	12.8%	32.1%	0	44.9%	50.89
Total Companies	26	26	26	78	4 df
% of row	33.3%	33.3%	33.3%	100%	2.35E-10
% of column	100%	100%	100%	100%	p-value.
% of total	33.3%	33.3%	33.3%		

Plot of Row % vs Depreciation Methods



(a) % of France of depreciation Method

(b) % of Germany of Depreciation Method

(c) % of UK of depreciation Method

10.39

$$\chi^2 = \sum_{\text{all}} \frac{(f_{ij} - \hat{E}_{ij})^2}{\hat{E}_{ij}}$$

$$= \frac{(69 - \frac{218 \times 200}{600})^2}{\frac{218 \times 200}{600}} + \frac{(97 - \frac{218 \times 200}{600})^2}{\frac{218 \times 200}{600}} + \frac{(52 - \frac{218 \times 200}{600})^2}{\frac{218 \times 200}{600}}$$

$$+ \frac{(101 - \frac{270 \times 200}{600})^2}{\frac{270 \times 200}{600}} + \frac{(93 - \frac{270 \times 200}{600})^2}{\frac{270 \times 200}{600}} + \frac{(76 - \frac{270 \times 200}{600})^2}{\frac{270 \times 200}{600}}$$

$$+ \frac{(30 - \frac{112 \times 200}{600})^2}{\frac{112 \times 200}{600}} + \frac{(10 - \frac{112 \times 200}{600})^2}{\frac{112 \times 200}{600}} + \frac{(72 - \frac{112 \times 200}{600})^2}{\frac{112 \times 200}{600}}$$

$$= 0.185 + 8.148 + 5.898 + 1.344 + 0.100 + 2.178 + (0.40 + 20.012) + 32.190 = 71.476$$

$\alpha = 0.01, df = (r-1)(c-1) = (3-1)(3-1) = 4$

$\chi^2_{4, 0.01}$ by p594, table A.10 查表得 13.2767.

因 $\chi^2 > \chi^2_{4, 0.01}$ ($71.476 > 13.2767$), 故

Reject H_0 : independent